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MATH 3D GEO VR

HANDBOOK

Mathematical models for teaching three-dimensional geometry using virtual reality

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UNIVERSITY OF SILESIA IN KATOWICE

HANDBOOK "Mathematical models for teaching three-dimensional geometry using virtual reality"

Created by the Math3DgeoVR consortium.

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the European Union

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HANDBOOK – content

About the project

The Math3DGeoVR project stands at the forefront of educational innovation, harnessing the transformative power of Virtual Reality (VR) to redefine the teaching and learning of geometry and spatial reasoning. This initiative is not an isolated endeavor, it joins a growing list of similar projects worldwide that recognize the immense potential of VR in education. These initiatives collectively underscore a paradigm shift in educational methodologies, leveraging technology to enhance learning experiences and outcomes.

Similar initiatives have demonstrated the effectiveness of VR in various educational settings. The other projects have aimed to make abstract mathematical concepts more tangible and comprehensible through immersive experiences. The success of such initiatives is backed by numerous scientific studies highlighting VR's positive impact on education. Research consistently shows that VR can significantly improve engagement, comprehension, and retention of complex subjects, making it a valuable tool in the educator's arsenal.

The Math3DGeoVR project, with its unique focus on geometric and spatial reasoning, seeks to cultivate these crucial skills that are integral to understanding and interacting with the world. From everyday tasks like solving puzzles and arranging furniture to professional applications in fields like engineering and architecture, these skills are indispensable. The project introduces an innovative approach to learning and teaching geometry, particularly in a multinational environment, enhancing the educational value of practicing and teaching these skills.

The objectives of the Math3DGeoVR project are comprehensive and multifaceted:

1. Developing students' geometric and spatial reasoning skills, with a special emphasis on two-dimensional plane geometry.
2. Increasing students' awareness of VR applications in mathematics.
3. Engaging students in the learning process through simulation and visualization offered by VR, providing an alternative to existing dynamic geometry software (DGS).
4. Making geometry training more enjoyable and immersive for both students and teachers.
5. Developing suitable VR activities for geometry teaching to strengthen students' spatial and geometric reasoning and better prepare them for the job market.
6. Enriching the mathematics curriculum in the field of three-dimensional geometry and supporting teachers with innovative tools and activities for classroom use.

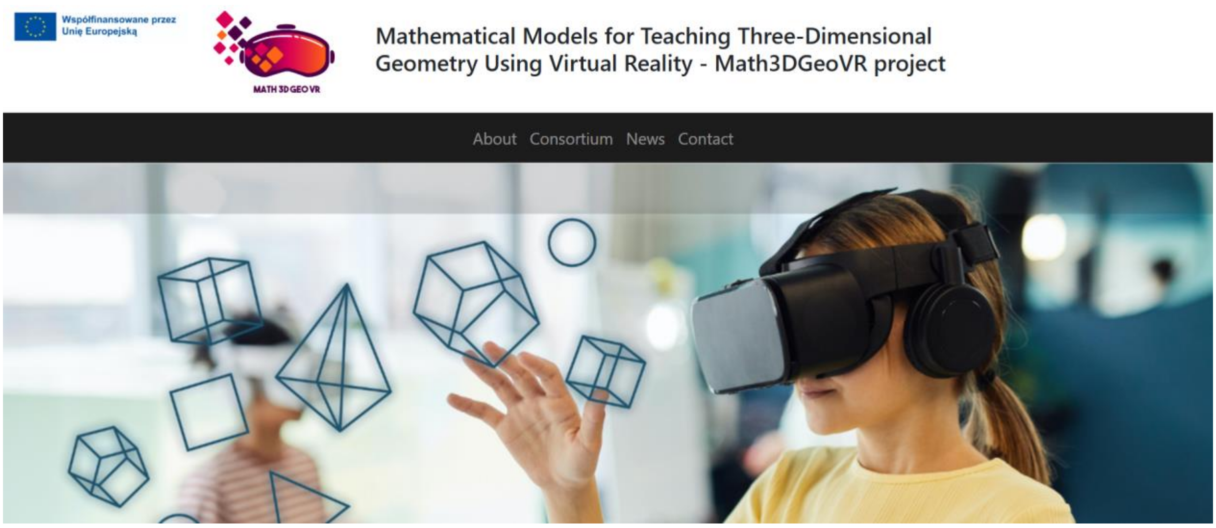
The anticipated outcomes of the Math3DGeoVR project are equally impressive:

1. Classroom-ready VR activities.
2. A suite of 13 teaching VR modules, each addressing different geometric and spatial problems.

The beneficiaries of this project are wide-ranging, encompassing students and academics from various fields, particularly those in engineering and teaching faculties. By integrating VR into the curriculum, **the Math3DGeoVR project is not just enhancing the learning of geometry, it is preparing students for a future where technology and education are inextricably linked, fostering skills that are essential in a rapidly evolving world.**

The Math3DgeoVR project logo

The Math3DgeoVR project website



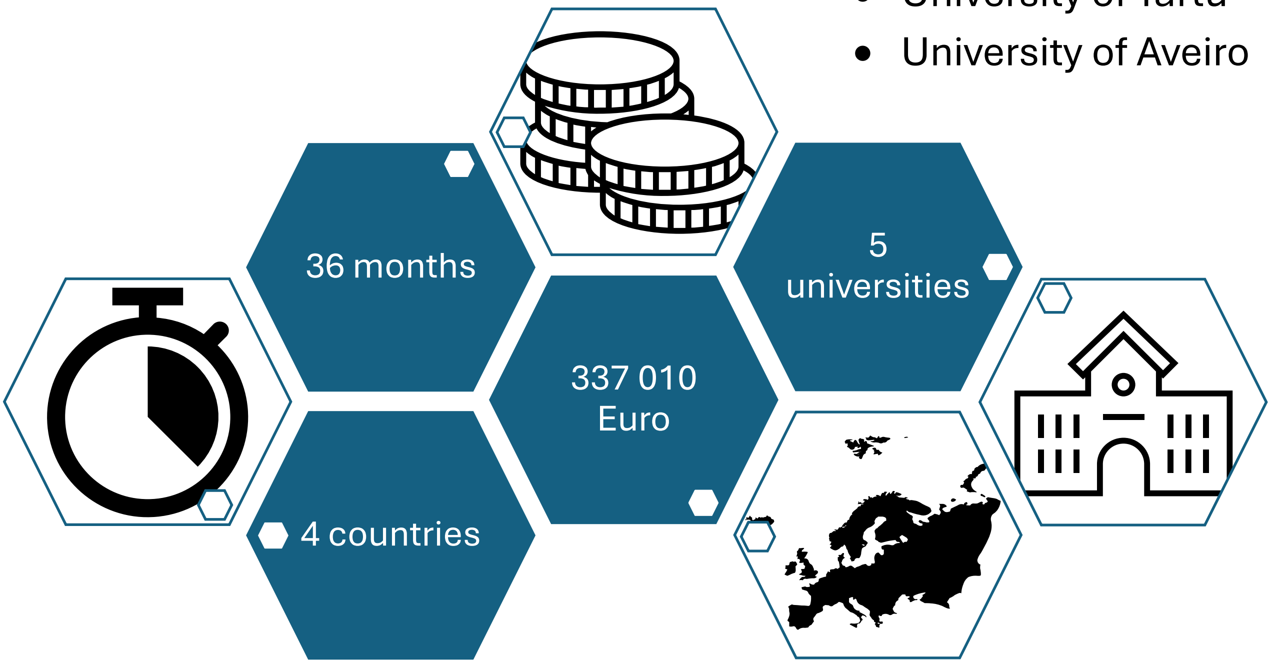
https://www.math3dgeovr.p.lodz.pl/

Math3DgeoVR project in numbers

Maths

in Higher Education

- Lodz University of Technology
- Silesia University
- University of Zilina
- University of Tartu
- University of Aveiro



- Poland
- Slovakia
- Estonia
- Portugal

Geometry & Virtual Reality

- 13 VR modules
- 26 lesson scenarios
- 1 complex learning book
- 6 Transnational Project Meetings
- 4 extra Technical Meetings
- 1 International Workshops (Summer School) for Students @UAVR
- 25 students participated in the international summer school
- 13 academic teachers participated in the international summer school
- 1 Final Project Seminar @ TUL
- Over 40 participants of the Final Project Seminar @ TUL
- 7 meetings with teachers/ educators in consortium countries
- over 100 participants of the dissemination events
- over 10 conferences where the project was presented

Introduction to Virtual Reality

What is Virtual Reality

Virtual Reality (VR) is a computer-generated simulation that allows users to immerse themselves in a three-dimensional (3D) environment, enabling virtual interaction. It provides a visual and sensory experience that closely mimics the real world [1]. The simulated environment can be based on either real or imagined scenarios, allowing users to engage with it as if they were in a comparable real-world situation. This way, the computer interface immerses users in experimental simulations, creating a more seamless connection between the computer and human participants [2], and making the experience feel real and unmediated. An example of a VR headset is presented in Figure 1 below.

Figure 1. A Meta Quest 2 virtual reality headset and controllers.
Source: https://en.wikipedia.org/wiki/Virtual_reality_headset#/media/File:Oculus_Quest_II_(50844634326).jpg

This way, VR systems share three main features: immersion, interaction, and a sense of presence. The degree to which each system allows the user to immerse, interact and experience the sense of presence can vary, therefore, offering different levels of user engagement and technological complexity. Understanding these differences is crucial for their application in different fields, including education and training. VR typically involves a head-mounted display that fully engages the user's senses, creating a sense of physical presence in a virtual environment. Immersive VR is found to enhance user experience by increasing presence, emotional engagement, and interactivity, while, as Vergara et al. [3] argue, the less immersive VR is, the less costly and, generally, more accessible. This is one of the reasons why such systems are noted for their practicality and ease of use, making them suitable for settings where quick setup and lower costs are priorities. More immersive VR systems, while offering a deeper level of engagement, often come with higher costs and more complex equipment requirements, which can be a barrier to widespread adoption [4]. VR systems have significant applications in education. The VR systems offering a more immersive experience have been shown to be particularly useful in increasing motivation and improving learning outcomes in complex subjects by providing a realistic and engaging environment. Conversely, less immersive VR systems are nowadays often used for more theoretical or diagrammatic content where higher levels of immersion are not essential [5].

According to Sugimoto et al. [6], several optical and visual factors influence how individuals perceive image quality and experience virtual environments (see Figure 2). Image clarity is determined by elements such as display resolution, the quality of optics, refresh rate, and field of view. Each of these aspects plays a critical role in shaping the overall visual experience in virtual worlds.

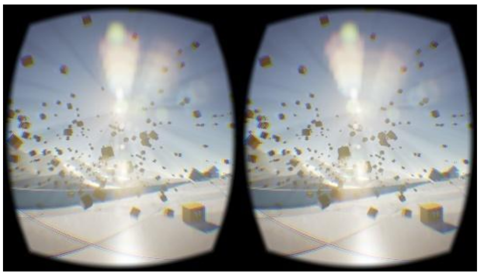


Figure 2. An image displayed by a VR headset, showing compensation for lens distortion and chromatic aberration.

Source:
https://en.wikipedia.org/wiki/Virtual_reality_headset#/media/File:Sample_screen_capture_of_Oculus_rift_development_kit_2_screen_buffer.jpg

Some applications of VR technology have already reached a high level of maturity, being widely adopted across industries such as gaming, healthcare, manufacturing, and education. For example, VR is commonly used for immersive gaming experiences, where its benefits are well established, and the technology is integrated into daily practices. On the other hand, other VR applications are still in their early stages of development. Areas such as virtual tourism, social interaction in VR spaces, and full-scale industrial training simulations are just beginning to explore the potential of VR, facing challenges like hardware limitations, scalability, and content development. As these technologies continue to evolve, they hold significant promise but require further refinement before they reach widespread adoption.

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History of Virtual Reality

Virtual Reality (VR) as a concept has roots in science fiction, with its popularization beginning with creating illusions of alternative realities. The advancements in immersive technologies allowed VR to evolve from a fictional idea to a practical tool used in various professional fields, including education. Virtual Reality has journeyed into a tangible technology that significantly impacts various sectors. This journey is not just about technological evolution but also a shift in conceptualizing how we interact with digital environments [1].

The inception of VR can be traced back to the 1950s with Morton Heilig's Sensorama, an early example of multisensory technology designed to make users feel as if they were "in" a film (Figure 3). The same year, Heilig was granted U.S. Patent 3,050,870 for a “Sensorama Simulator” and is widely credited with the first Virtual Reality stereoscopic head-mounted viewing equipment.

Figure 3. The Sensorama: One of the First Functioning Efforts in Virtual Reality.
Source: https://upload.wikimedia.org/wikipedia/commons/d/dc/Sensorama-morton-heilig-virtual-reality-headset.jpg

However, the concept of VR became more defined with Ivan Sutherland's introduction of the "Ultimate Display". In 1965, Sutherland, regarded as a pioneer of computer graphics, developed a device that could simulate reality to the point where one could not differentiate it from the actual world [2]. The computer display was equipped with a head position sensor that allowed the screen to adjust according to the user's head movements, often referred to as the precursor to Virtual Reality. As Sutherland once said: "The ultimate display would, of course, be a room within which the computer can control the existence of matter. A chair displayed in such a room would be good enough to sit in.

Handcuffs displayed in such a room would be confining, and a bullet displayed in such a room would be fatal" [3]. First showcased in 1968, the system earned the nickname "Sword of Damocles" due to its menacing mechanical apparatus hanging above the user (Figure 4), which was essential for tracking shifts in screen position to update the visuals accordingly. This conceptual groundwork laid the foundation for what VR would eventually become [1].

Figure 4. The "Ultimate Display" of Ivan Sutherland.
Source: https://cavrn.org/picturing-early-virtual-reality/

One of the most famous quotations of Sutherland's was:

"A display connected to a digital computer gives us a chance to gain familiarity with concepts not realizable in the physical world. It is a looking glass into a mathematical wonderland." (Sutherland, 1965)

Technological advancements in computer graphics, user interface hardware, and software frameworks throughout the 1980s and 1990s facilitated the development of VR platforms. The introduction of VR into the gaming industry, with systems like the Sega VR and Nintendo's Virtual Boy (Figure 5), brought the technology into mainstream awareness, though these early attempts were commercially unsuccessful due to technological limitations and high costs [1].

Figure 5. Nintendo's Virtual Boy game console with controller.
Source: https://en.wikipedia.org/wiki/Virtual_Boy#/media/File:Virtual-Boy-Set.jpg

Virtual Reality is regarded not just as a technological tool but as a pedagogical strategy that can lead to deeper learning and greater student engagement. The approach is holistic, involving careful design, development, and evaluation to integrate VR effectively into the educational process. This treatment of VR underscores its potential to revolutionize how subjects, particularly those involving complex visual and spatial information like trigonometry, are taught in modern classrooms. In the work of Rahmawati et al. [4], VR is approached as a transformative educational tool specifically designed to enhance students' understanding of trigonometry. The study highlights several key aspects of how VR is regarded and utilized [4]:

Innovative Learning Medium: VR is viewed as an innovative medium that goes beyond traditional teaching methods, which often rely on memorization. The VR environment enables a more interactive and engaging learning experience.

Enhancement of Conceptual Understanding: The main goal of employing VR in this context is to help students visualize and understand the abstract concepts of trigonometry. VR allows students to see and interact with mathematical shapes and angles in a dynamic way, which is difficult to achieve with standard textbook approaches.

Addressing Learning Challenges: The paper identifies specific challenges that students face in learning trigonometry, such as difficulty in visualizing concepts and applying memorized formulas to solve problems. VR is used to address these challenges by providing a more intuitive and experiential learning environment.

Virtual Reality creates a computer-simulated environment that can generate a three-dimensional space. This technology immerses users by replicating visual and other sensory experiences, making them feel as though they are part of the virtual world. VR allows for nearly realistic simulations and the construction of scenarios that are not feasible in the real world, thus reducing potential real-world risks [5].

As the affordability of VR has improved, it has become more widespread across various sectors, including medicine, architecture, entertainment, and tourism. This increasing accessibility has led to the development of VR applications by several major technology companies. In educational contexts, this study leverages VR to design digital teaching tools, blending traditional educational methods with innovative VR technology to foster a more engaging learning environment [5].

Virtual Reality is a technology that generates immersive, three-dimensional digital environments where users can interact as if they are part of that virtual world. The technology, which simulates both visual and other sensory details, enables experiences that are either difficult or impossible to engage in the real world. VR's potential extends beyond mere simulation, as it can create completely new scenarios, offering a safe and controlled setting for a variety of applications [6].

Despite its early inception in the mid-20th century with its first notable implementation being a flight simulator in 1966, VR's journey into mainstream commercial success has been fraught with challenges. Early ventures into VR-based entertainment, such as the arcade games by Virtuality Group in the 1990s and later the Nintendo Virtual Boy, met with limited success and were short-lived. These commercial setbacks, however, did not prevent VR from finding a receptive audience in other fields, particularly education [6].

Virtual Reality is increasingly recognized for its potential to transform traditional education systems. This transformative technology not only offers a novel method of engagement but also enhances the pedagogical process by providing immersive, interactive learning environments. This systematic review delves into how VR is being applied in educational settings and the motivations behind its use [7].

Virtual Reality's historical development from a conceptual oddity to a staple of technological advancement in education showcases its potential to transform how educational content is delivered and experienced. As VR technology continues to evolve, it promises to expand the boundaries of immersive and interactive learning, offering students a dynamic and engaging way to understand the complex world around them.

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Virtual Reality vs other emerging technologies

The defining differences between Augmented Reality and Virtual Reality

Immersive technologies comprise a range of innovations designed to merge virtual and physical realities, offering users a profound sense of immersion. Among the most prominent examples of these technologies are Augmented Reality (AR) and Virtual Reality (VR) [1].

Augmented Reality enhances the user's experience of the real world by overlaying interactive digital content- such as text, images, and videos- onto their physical surroundings. This integration allows users to interact with both real and virtual elements simultaneously, creating a rich, interactive experience that can be applied in various contexts, from gaming and education to retail and training [2]. Conversely, Virtual Reality offers an entirely artificial experience that transports users into a fully immersive three-dimensional environment. In this VR space, users are surrounded by digital visuals and sounds, effectively shutting out the physical world. This creates an opportunity for engaging simulations and experiences, such as virtual tours, training scenarios, and social interactions within a completely constructed digital realm [3]. Virtual Reality immerses users in a completely fabricated digital environment, while Augmented Reality (AR) overlays digital elements on the real world, enhancing real environments with virtual details without replacing the reality [4].

While VR and AR are often mentioned in the same breath due to their immersive technologies, they fundamentally differ in how they integrate with our perception of the real world. Understanding these differences is crucial for leveraging each technology's unique strengths, particularly in educational environments [4]. Virtual Reality immerses users in a synthetic environment where all visual and auditory experiences are generated by a computer. The primary objective of VR is to create a convincing, interactive world that users can explore and manipulate. In contrast, augmented reality superimposes computer-generated images and data over the real world, enhancing one's perception of reality rather than replacing it. AR can be seen as a blend of VR and real life, where digital and real-world elements coexist and interact in real time [4].

The technological underpinnings of VR and AR also differ significantly. VR typically requires headsets like Oculus Rift or HTC Vive, which block out the physical world and replace it with a digital one. These headsets are equipped with sensors that track the user's head movements and adjust the virtual perspective, accordingly, creating a deep sense of immersion. On the other hand, AR can be accessed with more simple devices such as smartphones and tablets or through more sophisticated wearable technologies like Microsoft's HoloLens, which overlay digital information onto the physical world through a transparent visor [4].

Together, AR and VR represent a significant evolution in how we interact with digital content, opening new avenues for communication, learning, and entertainment in our increasingly interconnected world.

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The Reality-Virtuality Continuum (1994)

Milgram and Kishino's [1] Reality-Virtuality Continuum, proposed in 1994, provides a comprehensive framework for understanding the spectrum of immersive experiences that blend the real and virtual worlds. This continuum ranges from completely real environments to entirely virtual ones, illustrating how various technologies facilitate interactions between these two realms.

At one end of the continuum lies the **Real Environment (RE)**, which represents the physical world as we know it. In this domain, all sensory experiences are genuine and unmediated, allowing individuals to engage fully with their surroundings [1].

Just beyond the real environment is **Augmented Reality (AR)**, which enhances the user's perception of the physical world by overlaying digital content—such as images, text, and videos—onto real-world scenes. AR creates a mixed experience where users can simultaneously interact with tangible objects and virtual elements, enriching their engagement with the environment. This technology has found applications across diverse fields, including gaming, education, healthcare, and retail, making it a versatile tool for enhancing real-world experiences [1].

Moving further along the continuum, we encounter **Mixed Reality (MR)**, which combines elements of both AR and Virtual Reality. In MR, users interact with virtual objects that coexist and react to real-world environments, enabling a more immersive experience. This technology allows users to manipulate digital content as if it were part of their physical space, fostering a seamless blend of real and virtual interactions. MR proves particularly valuable in training simulations, collaborative workspaces, and design applications, where the integration of digital tools into real-world tasks can significantly enhance productivity and creativity [1].

Situated between AR and VR is **Augmented Virtuality** (AV), characterized by a greater emphasis on virtual elements while still incorporating aspects of the real world. In AV, users primarily engage in a virtual environment, but real-world inputs—such as live video feeds or sensory data—are integrated into the experience. This approach enhances the sense of presence within virtual space while maintaining a connection to the physical environment [1] (Figure 6).

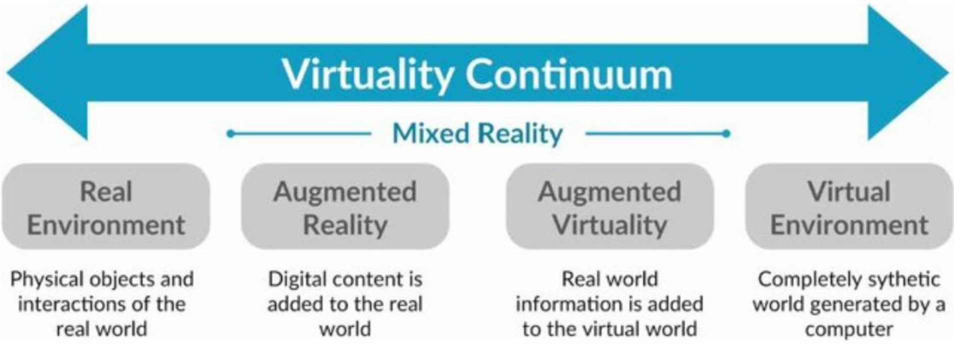

Figure 6. The Virtuality Continuum (Milgram & Kishino, 1994), From I. Giannios and D. G. Margounakis, "INSIDE: Using a Cubic Multisensory Controller for Interaction with a Mixed Reality Environment," *Research Anthology on Virtual Environments and Building the Metaverse*, vol. 5, no. 2, pp. 40-56, 2021. https://doi:10.4018/IJVAR.20210701.oa1

At the far end of the continuum lies **Virtual Reality (VR),** which offers a fully immersive experience where users are enclosed in a completely artificial environment. In VR, users engage with a digital world that can simulate real-life interactions and scenarios, often utilizing headsets and sensory equipment to create a multi-sensory experience [1]. This technology effectively disconnects users from their physical surroundings, allowing them to explore and interact within a fully constructed virtual realm. As can be seen in Figure 7, the user experience and interaction levels can substantially differ.

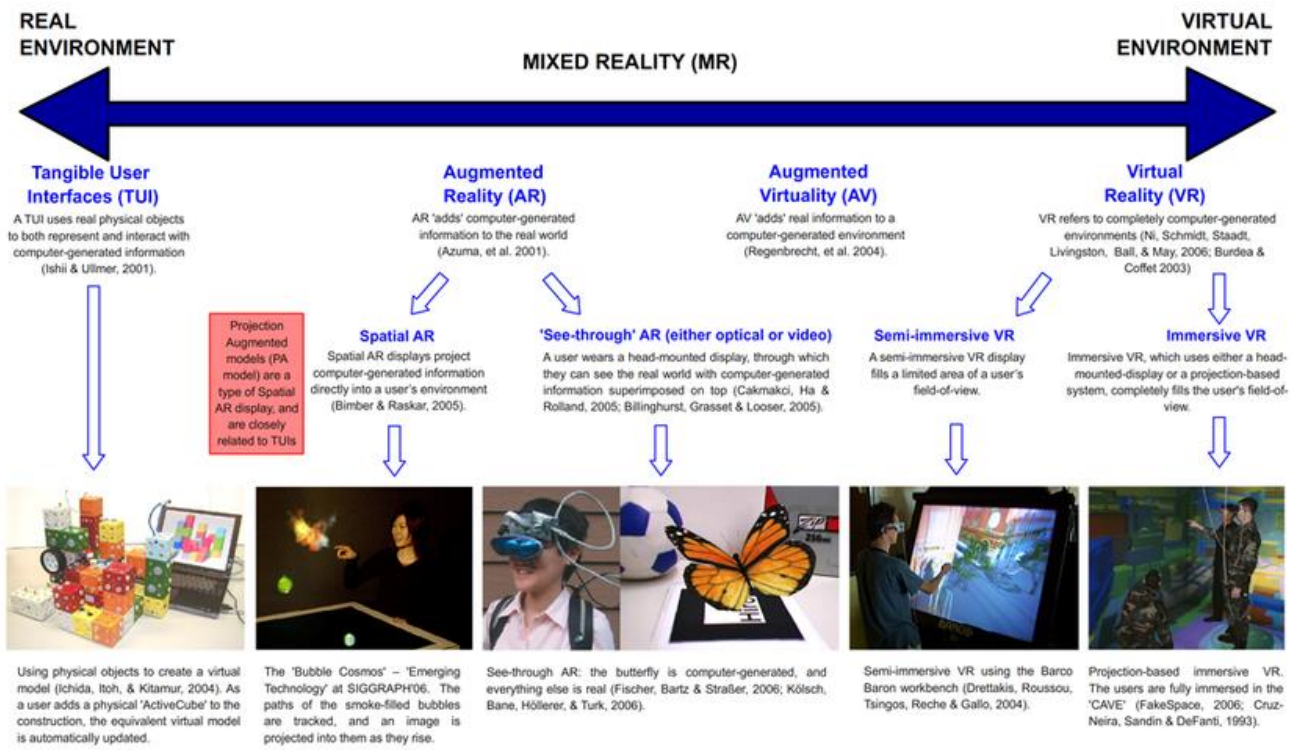

Figure 7. Reality-Virtuality Continuum. Source: M. S. Sidhu and L. C. Kang, "Interactive augmented reality environments for engineering with open systems," in *2011 IEEE Conference on Open Systems (ICOS2011)*, Langkawi, Malaysia, Sep. 25-28, 2011.

Overall, the Reality-Virtuality Continuum serves as a valuable reference for understanding the diverse experiences that immersive technologies can offer. By highlighting the varying degrees of immersion and interaction, this framework emphasizes the potential of these technologies to reshape our understanding of reality and enhance our engagement with both the physical and digital worlds [1].

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The X-Reality framework and more recent advances

The X-Reality framework significantly expands Milgram' and Kishino's (1994) understanding. The X-Reality framework proposed by [1] offers a comprehensive model for understanding and categorizing the various applications of Extended Reality (XR) technologies, which include Augmented Reality (AR), Virtual Reality (VR), and Mixed Reality (MR). Their framework emphasizes the unique characteristics and interactions of these technologies and highlights their implications across a range of sectors, such as education, entertainment, marketing, and healthcare.

At the core of the X-Reality framework is the outlining of a continuum of reality that spans from the physical world to fully immersive virtual environments. This spectrum encompasses several key dimensions. Physical reality represents the tangible environment that users interact with in their everyday lives. **Augmented Reality** enhances this experience by overlaying digital information onto the physical world, allowing users to engage with both digital and physical elements simultaneously. **Assisted Reality**, a subset of AR, provides digital elements that aid users in completing tasks while keeping them aware of their surroundings. **Mixed Reality** blends the physical and digital realms, enabling interactive experiences where virtual objects can interact with real-world environments. **Virtual Reality**, on the other hand, offers a fully immersive experience, isolating users from their physical surroundings and placing them in a computer-generated environment, which is commonly utilized in gaming, simulations, and training scenarios [1].

User interaction is another central aspect of the X-Reality framework [1]. The authors explore various modes of interaction, such as gesture-based controls, which enable users to manipulate virtual objects through physical movements. Voice commands allow for hands-free navigation within XR environments, while haptic feedback provides tactile sensations that enhance immersion and realism. By understanding how users engage with XR systems, developers can create more effective and engaging applications [1].

The context in which XR technologies are deployed plays a significant role in shaping user experiences. Different settings, such as retail, education, healthcare, and training, influence how users interact with XR. For instance, in educational contexts, Virtual Reality can foster immersive learning environments that deepen students' understanding of complex subjects, thereby enhancing educational outcomes [1].

Technological integration is also a crucial component of the X-Reality framework. It encourages exploration of how XR technologies can be seamlessly integrated with existing systems and processes. This includes ensuring compatibility with traditional software and hardware systems and considering scalability, which involves adapting XR applications to meet the needs of diverse user groups or organizational requirements [1].

Finally, the framework addresses the potential outcomes and benefits of implementing XR technologies. These may include improved learning outcomes resulting from immersive and interactive educational experiences, enhanced customer experiences through personalized engagement, and increased operational efficiencies by streamlining processes and facilitating better collaboration across teams [1].

The X-Reality framework developed by [1] serves as a valuable tool for analyzing and leveraging the diverse applications of XR technologies. By highlighting user interaction, contextual factors, and potential benefits, this framework provides insights for researchers and practitioners aiming to navigate the complexities and opportunities presented by the evolving landscape of extended reality. As these technologies continue to advance, the framework can guide organizations in effectively integrating XR tools to enhance user experiences and achieve strategic goals.

Therefore, Extended Reality covers a wide range of technologies, each providing different levels of immersion and interaction with digital content. VR immerses users in a completely artificial environment where the user's physical presence is simulated in a virtual world. AR overlays digital content onto the real world, augmenting elements of the environment with computer-generated inputs such as sounds, videos, or graphics. MR goes further by not only overlaying but also anchoring virtual objects to the real world and allowing for real-time interaction [2].

XR is an umbrella term that encompasses all immersive technologies including virtual reality, augmented reality, and mixed reality, as well as those that are yet to be developed. XR represents the full spectrum of experiences that combine elements of both digital and physical worlds or create completely immersive environments. Understanding XR is essential for grasping how these layered technologies can be integrated into various aspects of daily life, education, and professional fields [2].

Technologically, XR systems utilize a combination of advanced hardware and software, including sensors, headsets, AR glasses, mobile phones, and body-worn technology. These tools capture and analyze data from the physical world, integrating it with digital content to create seamless interactive experiences that blur the line between the virtual and real world [2].

References:

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Interesting examples of the use of Virtual Reality

Virtual Reality (VR) has already found significant applications in several fields. In **clinical psychology**, VR has been used in exposure therapy to treat anxiety disorders, phobias, and PTSD by safely immersing patients in controlled virtual environments to confront their fears. For example, patients with a fear of heights can gradually experience virtual heights in a controlled manner. In this regard, [1] argues that VR is an embodied technology with great clinical potential for experiential assessment and learning in anxiety disorders, pain management, and eating disorders. In the **healthcare sector**, VR is also applied in pain management by diverting patients' focus during painful medical procedures. For example, an efficacy study of a VR treatment in cancer patients has shown that a computer-generated virtual environment improved their emotional well-being and diminished cancer-related psychological symptoms [2].

Virtual Reality therapy shows promise in diagnosing and treating mental health disorders, with potential for widespread adoption in the healthcare field [3]. In **medical training**, VR is widely used in surgical training to provide realistic, hands-on practice for medical professionals. It allows surgeons to simulate complex procedures without risking patient safety. VR is also promising technology for **rehabilitation**, allowing users to interact with virtual environments and perform motor tasks, enhancing their recovery process [4]. By replicating real-life movements and situations, VR provides patients with a functional and motivating platform that encourages active participation in their therapy. One of the primary advantages is its ability to simulate purposeful tasks that mimic real-world activities, making therapy more relevant and enjoyable. Additionally, VR supports progressive task difficulty, enabling therapists to adjust exercises according to the patient's abilities and gradually increase complexity as their skills improve. This allows for individualized treatment plans that adapt dynamically based on the patient's progress. The motivational aspect of VR further aids rehabilitation by enhancing patient engagement, helping patients push through challenging tasks by making the therapy process more immersive and rewarding. Games and virtual challenges, often integrated into VR therapy, can boost adherence and patient motivation, which is critical in long-term rehabilitation programs. From the clinician's perspective, VR systems offer the benefit of tracking and documenting patient performance data in real-time. Automated tracking makes it easier for therapists to assess progress, monitor improvements, and adjust interventions as needed [5]. These features combined make VR a powerful tool for motor recovery, enhancing both the experience for patients and the clinical insights for therapists.

From a different angle, VR holds immense potential for transforming industry, such as the **tourism industry**, particularly for individuals with limited mobility, financial constraints, or other restrictions (e.g., age or physical conditions). VR can offer immersive travel experiences that allow people to explore destinations around the world without leaving their homes. For older adults or those with disabilities, VR provides a way to engage with places they might never be able to visit in person due to physical limitations. Additionally, during times of restricted travel, such as during the Covid-19 pandemic, VR served as a valuable alternative for experiencing new places. VR allows users to interact with detailed, virtual recreations of famous landmarks, museums, or nature reserves, providing a sensory-rich experience that goes beyond typical video or photographic content. These virtual tours can also be a steppingstone allowing individuals to explore potential destinations before committing to a costly trip. In their study, Canio et al. [6] have shown that immersion and presence in VR tours directly impact enjoyment and satisfaction, which in turn influence visitors' physical visit intention. Furthermore, VR can be an educational tool in tourism, offering historical, cultural, and environmental insights through immersive storytelling. As Lee et al. [7] have found, interactivity and vividness in VR positively influence customers' information search and sharing behaviors. By breaking down barriers to exploration, VR is making travel more inclusive and accessible for a broader population. To conclude, VR has potential applications in tourism planning, management, entertainment, education, accessibility, and heritage preservation, but acceptance of virtual experiences depends on tourists' attitudes and constraints [8].

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VR in Education

Rationale for Using Virtual Reality in Education

The integration of Virtual Reality (VR) into educational settings is driven by several reasons. At the core of these motivations is the goal to enhance educational outcomes through increased student engagement and lead to a better retention of information. This holds true even for students who might find traditional learning environments uninspiring [1].

First, VR can foster **enhanced engagement and motivation** [1]. VR technology is a web-based learning environment that enhances 3D graphics and visualization, promoting participation and student learning [2, 3]. By creating such immersive and interactive experiences, VR can capture students' attention, elicit positive emotions, and make learning more enjoyable. Some studies have shown that VR can increase student motivation, reduce absenteeism, and improve overall satisfaction with the learning process. To exemplify, a study conducted among college students explored how interactive virtual reality enhances learning outcomes by investigating psychological and emotional factors. The findings suggest that enjoyable emotions experienced by learners in VR learning environments m

mediate the relationship between VR features and perceived learning outcomes [4]. In another study, researchers sought to determine the extent to which Extended Reality (XR)-immersive labs would change the students’ perception and motivation toward learning in kinesiology courses. It was found that such activities enhanced students' motivation to learn by enabling them to interact with visual content and engage in immersive experiences [5]. The research team led by Deng [6], investigated the effectiveness of VR in animation education and explored the use of VR in animation art experimental teaching. These authors found that VR in animation art experimental teaching increased students' interest and enthusiasm, with 96.25% of them favoring the system.

Second, VR **can provide personalized learning experiences** [1]. By adapting content to individual student needs and preferences, VR can help address learning gaps, increase engagement, and improve overall learning outcomes. For example, students with learning disabilities can benefit from VR simulations that provide visual aids and support, while advanced students can explore more challenging content as found by Sung et al. [7]. Specifically, they conducted a study examining the effectiveness of VR in personalized learning for students with learning disabilities. Participants were randomly assigned to either a VR group or a control group. The VR group used VR-based simulations to learn mathematical concepts, while the control group used traditional teaching methods. Results showed that students in the VR group had significantly higher levels of academic performance, self-efficacy, and engagement compared to the control group. In another study, provides valuable insights into how VR can enhance learning experiences for students with learning disabilities [7].

Third, VR **can facilitate experiential learning** [1]. By allowing students to actively engage with the material and explore concepts in a hands-on manner, VR can promote deeper understanding and retention of knowledge. A recent study that examined the impact of VR experiential learning on undergraduate students' knowledge and evaluation skills related to assistive technology for older adults and individuals with disabilities indicated that VR-based experiential learning significantly enhanced students' understanding and practical skills in this area [8]. Moreover, Mercer et al. [9] conducted a review of the literature on the use of VR in science education. They found that VR can provide students with unique learning experiences that can enhance their understanding of scientific concepts and develop their problem-solving skills. For instance, students can virtually visit historical sites, conduct scientific experiments, or practice medical procedures in a simulated setting [9].

Fourth, VR **can improve collaboration and communication** [1]. By creating shared virtual spaces where students can work together on projects, communicate with peers, and develop problem-solving skills, VR can enhance social and emotional learning, what can prepare students for the collaborative nature of the modern workplace. In this regard, some research has evidence has been found illustrating how VR can play a critical role in improving collaboration, communication, and social-emotional learning important in the job market. In their systematic literature review, Asad et al. [10] review and synthesize studies that explore the role of VR in educational settings and how it can enhance different aspects of learning, including communication and collaboration skills. The authors discuss how VR can simulate real-world scenarios that require communication, thereby helping students develop their social, verbal, and non-verbal communication skills. VR is highlighted as a tool that can foster collaborative learning, enabling students to practice communication in controlled, but realistic settings, which can lead to improved engagement, more confidence, and better interpersonal interactions. By immersing students in virtual environments, VR provides opportunities for practice in problem-solving, teamwork, and communication in realistic contexts (e.g., public speaking, customer service, and conflict resolution). This is particularly useful for students who need to enhance their ability to work in teams, as the study suggests that VR improves both individual and group communication abilities. The review suggests that VR can prepare students for the collaborative nature of the modern workplace. As many industries increasingly rely on teamwork, virtual environments help students develop the communication skills and problem-solving abilities needed to thrive in such settings. In particular, VR enhances social-emotional learning by encouraging students to engage in empathetic communication, handle group dynamics, and solve problems together [10].

Another review [11] systematically analyzed existing literature on the use of VR in collaborative learning. The work aimed to explore how VR can support and enhance collaborative learning environments. The authors found that VR significantly improves engagement and motivation among learners, increasing their participation in collaborative activities. Additionally, VR supports distance learning by enabling collaboration across geographical boundaries, fostering teamwork and communication skills. The review also highlighted the potential of VR to create interdisciplinary spaces that enhance learning by allowing diverse perspectives. However, while VR was found to align well with various collaborative learning paradigms, the authors emphasized the need for further research to develop pedagogical frameworks and best practices for VR-based collaborative learning. Ultimately, the study concluded that VR is a powerful tool for enhancing collaborative learning experiences but requires more structured integration into educational settings [11].

One interesting tool developed in the spectrum of collaboration is the CLEV-R (Collaborative Learning Environment with Virtual Reality), whose authors aimed to enhance e-learning experiences by integrating VR into collaborative learning environments [12]. CLEV-R is a web-based application that combines VR and multimedia to deliver learning materials in an engaging manner. It includes communication tools designed to support collaboration among students. With stimulating experiences and adaptable features that facilitate interaction and collaboration among learners, an initial effectiveness evaluation of CLEV-R in supporting collaborative learning indicated that the system successfully facilitated communication and collaboration among students [12].

The integration of VR into educational frameworks offer promising enhancements to traditional teaching methodologies. VR can transcend geographical barriers, allowing students to explore global issues and cultural experiences from their classrooms. This not only broadens their understanding but also fosters empathy and a better appreciation of global diversity. As VR technology continues to evolve, its integration into educational systems is expected to expand, further enriching the learning and teaching processes [1].

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Virtual Reality as means to redefine the teaching and learning process

One of the primary ways Virtual Reality (VR) can redefine classroom instruction is by enhancing student **commitment** to the learning process. For example, previous studies have shown that VR can increase student **attention** [e.g., 1] **and retention rates** [e.g., 2]. The study of Essoe et al. [3] examined how VR environments can impact learning and retention. The authors hypothesized that immersive VR environments, which provide rich sensory experiences, could serve as powerful tools for enhancing learning. They specifically explored how distinctive VR environments that are contextually rich and varied could facilitate better retention by reinstating the mental context in which learning occurred and found that participants who experienced distinctive and immersive VR environments showed improved learning and memory retention compared to those who learned through more traditional methods [3]. By immersing students in contextually rich environments, VR not only **promotes deeper engagement and understanding** but also enhances the ability to recall information and **makes learning more memorable**, offering a promising alternative to traditional learning methods.

The incorporation of VR in education has opened new avenues for representing and interacting with complex **scientific concepts**, providing an immersive platform that enhances understanding through virtual manipulation and dynamic visualization [4]. Teachers can use VR to introduce new concepts, provide visual aids, or facilitate group activities, what helps make learning more engaging and effective for students, and VR can be used to supplement traditional classroom instruction, providing students with **additional opportunities to learn and explore**. Therefore, VR seems to have the potential to transform the teaching-learning process, however, its mass adoption among faculty is yet to occur [5].

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Virtual Reality in different fields of education

In recent years, the focus of computer-assisted learning has shifted from promoting passive participation to encouraging active, self-directed learning. This change aligns with constructivist learning theories, which advocate for students to take control of their learning by identifying, synthesizing, and correcting misconceptions. Cognitive theories have further driven the integration of interactive simulations in education, enhancing engagement. Modern educational technology now goes beyond simply using computers or software, integrating various tools to enrich teaching methods and improve learning outcomes [1].

Integration of Virtual Reality (VR) into education aligns with the rise of immersive learning, where students engage in an interactive environment that simulates real-world scenarios. In the educational context, VR's potential was quickly recognized for its ability to provide immersive learning environments. By simulating real-world scenarios, VR has the capability to enhance understanding through experience rather than through passive observation. This application has been particularly valuable in fields that benefit from experiential learning, such as medicine, aviation, and engineering [2].

In academia, VR technology began to be seriously considered as a pedagogical tool in the late 1990s and early 2000s. The evolution of VR in education was marked by the introduction of more accessible and cost-effective technology, such as the Oculus Rift (Figure 8), which was initially funded through a kick-starter campaign in 2012. This headset was a game-changer, providing high-quality immersive experiences at a consumer-friendly price point, thus opening new opportunities for its use in educational settings [2].

Figure 8. The Oculus Rift CV1 (Consumer Version 1), released in 2016.
Source: https://en.wikipedia.org/wiki/Oculus_Rift#/media/File:Oculus-Rift-CV1-Headset-Front.jpg

The application of Virtual Reality in education spans a diverse range of uses, each tailored to enhance learning by leveraging the immersive capabilities of technology. One of the primary applications of VR is in the simulation of complex environments and scenarios that are either impossible, risky, or too expensive to replicate in real life. For instance, history students can visit ancient civilizations, which provides a depth of learning that textbooks cannot achieve [3]. Moreover, VR is employed in skills training where practical, hands-on experience is crucial but difficult to facilitate in a traditional classroom. This includes everything from surgical procedures in medical schools to crisis management drills in emergency responder training. VR enables students to practice and hone their skills in a safe, controlled, and repeatable environment [3]. VR has been widely used to develop virtual laboratories where students can safely conduct experiments in a controlled and repeatable environment. This approach is particularly beneficial in subjects like chemistry and physics, where real-world experiments may be expensive or pose safety risks. Additionally, VR enhances the understanding of abstract concepts in subjects such as mathematics and physics, offering a more concrete way to engage with complex theories [2].

The integration of Virtual Reality in mathematics education represents a transformative shift in how educational content is delivered and experienced by students. This technology's specific use in mathematics aims to enhance comprehension, engagement, and interaction with complex mathematical concepts through immersive visualization and interaction.

In mathematics education, particularly at higher levels, VR allows for the visualization of abstract concepts and complex functions, providing a three-dimensional space to explore and manipulate mathematical elements [2]. The review of Lai and Cheong [2] discusses the application of VR across basic, secondary, and higher education, illustrating various pedagogical strategies and technological integrations tailored to each educational level, while analyzing different studies and frameworks to demonstrate VR’s effectiveness and limitations in enhancing mathematical understanding and engagement. The authors provide a comprehensive overview of how VR is applied specifically in the context of mathematics education, highlighting its potential to bridge the gap between abstract mathematical concepts and their practical applications, specifically:

Visualizing Abstract Concepts: VR uniquely enables the visualization of complex mathematical forms and structures, which are often difficult to represent through traditional 2D methods. For example, in multivariable calculus or geometry, VR can allow students to manipulate and explore 3D mathematical models, enhancing their understanding of shapes, volumes, and dynamics in a way that textbooks cannot.

Interactive Learning Environments: The review points out that VR creates interactive learning environments where students can engage directly with mathematical problems and simulations. This interactivity promotes active learning, which is crucial in understanding and applying mathematical theories effectively.

Enhancing Engagement and Motivation: According to the findings in the paper, VR has shown potential in increasing student motivation and engagement. By turning learning into an immersive and enjoyable experience, VR can help overcome the traditional challenges associated with mathematics education, such as high cognitive load and low engagement.

Simulation of Real-World Applications: VR can simulate real-world scenarios where mathematical concepts are applied, providing students with practical contexts and problems. This application not only helps in understanding the relevance of mathematical theories but also in developing problem-solving skills.

A study conducted by Hsu [1], delves into the integration of Virtual Reality within high school mathematics, focusing on the system of linear equations in three unknowns. The primary aim is to explore how VR can enhance learning motivation and effectiveness when incorporated into mathematical education. This integration signifies a shift towards immersive learning environments, where digital technology plays a central role in educational settings. Hsu's research collaborates with high school mathematics teachers to design VR-based tutorial materials that not only aid in learning but also make the process more engaging and interactive. The utilization of VR allows students to visualize and interact with mathematical concepts in three-dimensional space, potentially increasing their interest and understanding. The study posits several research questions aimed at validating the effectiveness of VR in teaching complex mathematical concepts. These include inquiries into VR's suitability for teaching linear equations, its impact on learning motivation, and its effect on learning effectiveness. The hypotheses suggest that VR not only enhances understanding of these concepts but also addresses cognitive errors encountered often with traditional 2D teaching materials.

Employing a quasi-experimental design, the study uses VR tutorial materials as intervention tools in teaching linear equations. The methodology involves pre-tests and post-tests using the ARCS model and Bloom’s taxonomy to measure shifts in student motivation and cognitive understanding. Initial findings indicate an improvement in learning motivation and effectiveness among students exposed to VR teaching methods. The VR environment allows students to engage with the material in ways that are not possible in traditional learning scenarios, such as manipulating and observing geometric shapes and equations in real-time and three-dimensional space. The study concludes that VR technology significantly supports the teaching of complex mathematical concepts, enhancing both student engagement and understanding. It recommends further research into the use of VR in different educational contexts and subjects to fully exploit its potential in enhancing learning outcomes [1].

A study of Rahmawati et al. [4] has addressed the challenges students face in understanding trigonometry. The authors argue that traditional methods, which often rely heavily on memorization, do not adequately support students in grasping the conceptual foundations of trigonometry. As a solution, they proposed the use of VR as an innovative educational tool designed to enhance learning outcomes in high school mathematics. The research utilizes the ADDIE model, a systematic approach to instructional design that includes Analysis, Design, Development, Implementation, and Evaluation, which served as the framework for developing the VR-based educational media. The process began with a detailed analysis of existing learning challenges and educational needs, followed by the design and development of the VR content tailored specifically for trigonometry education. The effectiveness of this media was then evaluated through both expert validation and classroom implementation. The findings from the study highlight the effectiveness of the VR learning media in enhancing students' understanding of trigonometry. The media was validated by both material and media experts, receiving high marks for feasibility, content accuracy, and user engagement. Specifically, material validation focused on the substance, language use, and the relevance of the content to learning objectives, while media validation assessed the general appearance, presentation, language feasibility, and visual communication of the VR application. Through the application of VR, students were able to engage with trigonometry in a more interactive and immersive environment. This method proved particularly effective in helping students visualize and understand abstract concepts through dynamic and interactive content. The VR media allowed for a visual and experiential learning experience that traditional methods could not offer, making the learning process not only more engaging but also more effective in terms of long-term retention and understanding. The study concluded that VR-based learning media significantly improved the understanding of trigonometry among high school students. The VR environment provided a unique opportunity for students to explore and interact with mathematical concepts in a way that demystifies complex topics and enhances their learning experience [4].

The use of VR in education has been the subject of a number of comprehensive analyses aimed at evaluating its effectiveness in enhancing learning outcomes and overall educational experiences. Pellas et al. [5] have conducted a systematic review that has found that immersive VR applications can potentially improve students' learning performance in K-12 and higher education settings, but require careful consideration of user experience, usability issues, and learning outcomes. [5]. The work of Kania and

Kusumah [6] provides a comprehensive review and analysis of the literature on the role of VR in mathematics education over a significant period. This analysis is crucial for understanding how VR technology has evolved and been adopted in the educational sector, specifically in teaching mathematics. The research covered in the article indicates a robust growth in VR applications in mathematics teaching from 1994 to 2022. This period saw the development and refinement of VR technologies that have gradually been integrated into educational practices to enhance the teaching and learning of complex mathematical concepts. Studies included in the bibliometric analysis demonstrate that VR can significantly enhance learning outcomes in mathematics by providing students with visual and interactive experiences that are not possible through traditional teaching methods. VR environments allow students to manipulate mathematical models and visualize abstract theories in three dimensions, thus fostering a deeper understanding of the subject matter. According to the findings highlighted in the review, VR has the potential to increase student engagement and motivation. The immersive nature of VR makes learning more engaging and enjoyable, which can lead to better attendance rates, increased attention spans, and higher overall satisfaction with the learning process [6]. Their work provides some practical examples and applications of the use of VR in math education, specifically:

Virtual Manipulatives: VR can create interactive tools that students can manipulate to understand geometry, algebra, and calculus concepts better. These tools provide a hands-on experience that can make abstract concepts more concrete.

Simulated Real-World Problems: Through VR, students can solve real-world mathematical problems in a simulated environment. This method not only makes learning more relevant but also enhances problem-solving skills by applying mathematical theories to practical scenarios.

One meta-analysis of Virtual Reality training programs has found that VR training programs produce better outcomes than tested alternatives, with task-technology fit and aspects of the research design influencing results [7]. Another meta-analysis evaluated methodological trends in behavioral research on VR with respect to data collection practices, statistical reporting, and data availability. In line with this goal, the authors conducted a meta-scientific analysis of 61 articles encompassing a total of 1122 statistical tests. Results offered important findings suggesting that VR research needs increased transparency, better statistical power, and more careful reporting of statistical outcomes to enhance methodological rigor and reproducibility [8].

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Challenges related with the use of VR in educational settings

Virtual Reality (VR) software development has progressed significantly. In educational settings, VR has been explored for its potential to increase engagement and enhance the learning experience. Studies have shown that VR can lead to benefits such as increased time spent on tasks, heightened enjoyment, and improved motivation among students. These positive outcomes highlight VR's capability to transform traditional educational models and offer more dynamic and interactive learning experiences.

Despite these advantages, the widespread integration of VR in education has been slow, owing to technological and cost barriers, which have restricted its full-scale adoption in mainstream educational practices [1]. In this regard, Kania and Kusumah [2] argue that main challenges of the integration of VR in mathematics education include the high cost of VR hardware and software, the need for significant teacher training, and the potential for technological distractions. Moreover, there is a need for more empirical research to validate the effectiveness of VR in improving mathematical skills over traditional methods. The insights from their study provide a roadmap for future exploration and integration of VR technologies in educational practices [2]. Similarly, Lai and Cheong [3] point out the high cost of VR equipment as well as the possibility for technological distractions. They call for careful consideration of when and how VR is implemented in the educational process. In their systematic review, Kavanagh et al. [1] have identified several limitations and challenges in using VR in education. Key issues include:

High Costs: The expense of purchasing and setting VR hardware and software presents a significant barrier for many educational institutions, especially those with limited resources. The cost must account for not only the initial purchase of hardware and software, but ongoing costs such as maintenance, support, and training.

Technical Issues: VR systems require strong infrastructure (such as powerful computers, adequate space). Moreover, some users experience motion sickness, system glitches, and other technical challenges, which can disrupt learning experiences.

Usability and Interactivity: While VR can provide immersive experiences, some systems lack intuitive interaction methods, such as gesture controls or seamless user interfaces. This reduces the ease with which students can engage with the content.

Limited Pedagogical Alignment: There is often a lack of well-defined pedagogical frameworks for integrating VR into curricula. Many VR applications are not specifically designed for educational purposes, limiting students' engagement and, thus, their effectiveness. Educators may find it difficult to align VR activities with established learning outcomes, and there is a need for more research into how VR can best support different teaching methods, such as constructivist or collaborative learning.

Accessibility and Equity: Access to VR technology may create a divide between institutions and students who can afford it and those who cannot, reinforcing existing inequalities in educational opportunities.

The integration of Virtual Reality specifically into STEM education presents exciting opportunities but also faces significant challenges, particularly regarding time and cost limitations:

Cost Limitations: Virtual Reality technology, while becoming more affordable, still poses substantial cost barriers for widespread adoption in educational settings. The development of low-cost VR solutions has been a focus of recent studies, attempting to make this technology more accessible for classroom instruction. For instance, efforts to create affordable 3D immersive applications for engineering education highlight ongoing challenges and advancements in reducing costs [4].

Time Limitations: The implementation of VR in education requires significant time investment both in terms of setup and integration into curricula. Educators must allocate time to learn the technology, develop VR-based lessons, and integrate these into existing teaching frameworks. Studies suggest that while VR can enhance learning of complex and abstract concepts, the time required to create and implement VR experiences can be substantial [5].

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Current and future opportunities of the use of VR in educational settings

The uses of Virtual Reality (VR) and Augmented Reality (AR) differ due to their unique technological characteristics. VR is highly effective in creating fully immersive, controlled environments for disciplines such as medical training or historical exploration, enabling students to simulate surgeries or visit ancient locations. In architecture, VR allows users to virtually explore buildings before they are constructed. On the other hand, AR excels in enhancing real-world interactions by overlaying digital elements. In science education, for example, AR can project molecular structures onto physical surfaces, enabling students to visualize and manipulate complex data. Similarly, in mathematics, AR can superimpose geometric shapes or graphs onto the real environment, making abstract ideas more interactive and easier to grasp. As AR technology continues to develop, its potential in educational settings is expanding. Future research could explore how AR tools can be integrated into textbooks and educational materials to create dynamic, interactive learning experiences that adjust to the pace and progress of the student [1]. Additionally, as AR devices become more capable and less obtrusive, they could become a common tool in everyday educational practices. Further research is also needed to understand the cognitive impacts of learning with AR versus VR. Studies could examine how these technologies affect knowledge retention, student engagement, and the development of spatial reasoning skills. Moreover, the potential for personalized learning experiences through AR could be explored, examining how real-time data from students' interactions with AR content could be used to adapt instructional methods and materials to better meet individual learning needs [1]. In educational settings, Extended Reality (XR creates flexible learning environments tailored to the lesson's context. For example, it can immerse students in scientific concepts by visualizing complex phenomena that are difficult to replicate in traditional classrooms, such as astronomical events or molecular structures. In cultural education, XR enables students to explore historical sites or artifacts in detail, improving engagement and retention. For instance, XR can bring historical events or ancient civilizations to life, allowing learners to interact with the past. In environmental science, XR simulates ecosystems, illustrating ecological processes in real-time and enhancing students' understanding of environmental conservation. XR provides a unified approach to immersive technologies, combining VR, AR, and Mixed Reality (MR) to create adaptive, interactive learning experiences that exceed the capabilities of any single technology. As XR evolves, it holds the potential to revolutionize education by personalizing content and making lessons more interactive. Further research could focus on how best to use XR for fostering student engagement, critical thinking, and problem-solving. Moreover, integrating XR with AI could tailor educational experiences to each student's needs, offering personalized and adaptable learning in real-time [1]. As technology becomes more integrated into mainstream education, research focusing on its pedagogical impact becomes increasingly vital. Studies could explore how VR can be optimized to improve learning outcomes, the cognitive load involved with VR training, and the long-term retention of information learned via immersive experiences [1].

Recommendations from the study of Hsu [2] include:

Further Development: There is a recommendation for ongoing improvement and optimization of VR teaching aids to be seamlessly integrated with traditional teaching methods. This integration aims to complement and augment the capabilities of both teaching staff and instructional materials, thus providing a richer and more effective learning environment for students.

Broader Application: The study suggests expanding the application of VR technology to other units and subjects within the high school curriculum. This would allow for a more comprehensive evaluation of VR's impact across various educational domains.

Comparative Studies: Future research should also consider comparative studies that assess learning motivations and outcomes before and after the introduction of VR-assisted teaching methods. This could provide clearer insights into the specific benefits and limitations of VR in educational settings.

Scope Expansion: It is recommended to extend the research to cover more areas of the high school curriculum. This would help in determining the versatility and adaptability of VR technologies across different subjects and educational challenges.

Furthermore, the development of VR technology could focus on enhancing interactivity and reducing the occurrence of cybersickness, which remains a barrier to the widespread adoption of VR in educational settings. Future technological improvements could also explore the integration of artificial intelligence to tailor learning experiences to individual needs, potentially revolutionizing personalized education [1].

The future of VR in education and mathematics education in particular looks promising, with ongoing advancements in technology potentially lowering costs and increasing accessibility. The full potential of VR in education will be realized through continued technological development, pedagogical integration, and empirical research. To achieve this goal, further work is suggested in the following areas [1]:

Effectiveness of VR Learning: Continuous research is needed to evaluate the long-term effectiveness of VR in improving mathematical skills and whether it significantly impacts learning outcomes compared to traditional methods.

Curriculum Integration: Developing best practices for integrating VR into existing mathematics curricula and determining the optimal balance between virtual and traditional learning experiences.

Teacher Training and Technological Fluency: As VR becomes more common in educational settings, teacher training programs must adapt to equip educators with the necessary skills to effectively implement VR technology in their teaching.

References:

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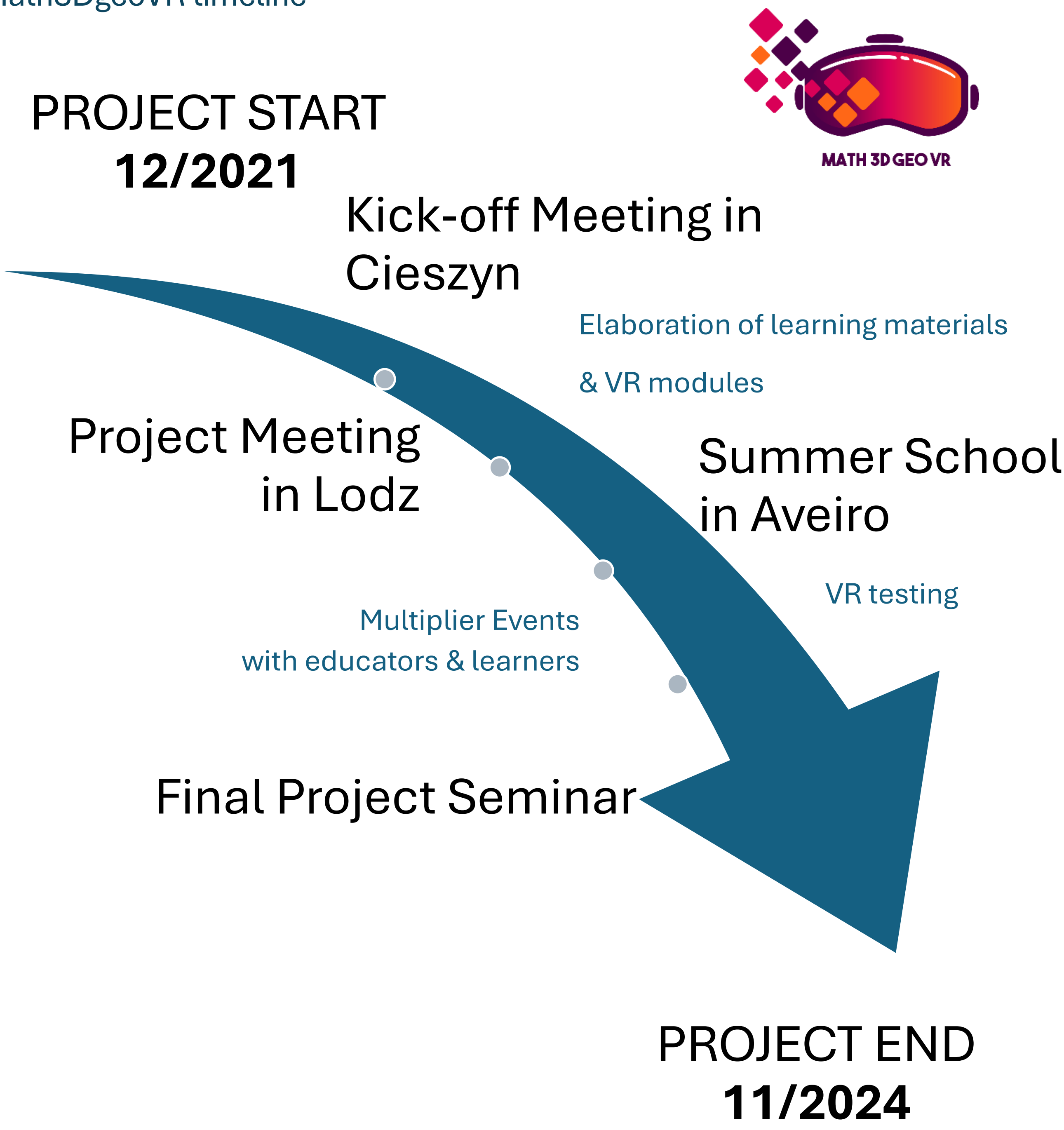
VR in Math3DgeoVR in Action

The **Math3DGeoVR** project has introduced an innovative dimension to teaching and learning geometry and spatial reasoning through Virtual Reality (VR). With a series of engaging activities and workshops, this project has bridged the gap between theoretical mathematics and practical application, fostering a deeper understanding of complex concepts in a highly interactive manner. This chapter highlights the project's practical implementation, with an emphasis on the 2024 Summer School in Aveiro, multiplier events, and VR modules tested during the initiative.

Math3DgeoVR timeline



Math3DgeoVR SNAPSHOTS

Kick-off Meeting in Cieszyn, Poland

Summer School in Aveiro, Portugal


IS TO QUIT TALKING &
BEGIN DOING
TECH + TOURISM
LEARNING TO BE

Project Meeting @ TUL

Final Project Seminar in Lodz, Poland

Final Project Meeting @ TUL

Summer School in Aveiro 2024

The Summer School, hosted by the University of Aveiro from June 17-21, 2024, was a cornerstone event of the Math3DGeoVR project. It brought together 25 students from five institutions, including Lodz University of Technology, University of Silesia, University of Aveiro, University of Žilina, and Tartu Ülikool, as well as 13 academic staff members.

The week-long intensive program focused on exploring mathematical relationships such as congruence, symmetry, reflection, rotation, expansion, and contraction through VR. Participants engaged in six core modules:

- Trajectory: Trigonometric functions in 3D space.
- Angles in Prisms.
- Angles in Pyramids.
- Non-Euclidean Geometry.
- Maxima and Minima of Functions.
- Systems of Linear Equations.

The sessions were guided by

- Dr. Jacek Stańdo, Prof. Lodz University of Technology,
- Dr. Tomasz Kopczyński from the University of Silesia,
- and Adam Nowak, MSc, from Lodz University of Technology.

In addition to VR modules, participants engaged in workshops on creativity, entrepreneurship, and communication, led by Prof. Ewa Korzeniewska, Dr. Eng. Agnieszka Pietras, and Dr. Eng. Iwona Staniec from Lodz University of Technology. The closing ceremony, summarized by Prof. Nina Szczygiel, highlighted the exceptional energy and innovative spirit observed during the competitive sessions.

The event was also observed by distinguished academics, including Dr. Sylwia Kania from the University of Silesia, Prof. Mária Kúdelníčková and Prof. Beatrix Bačová from the University of Žilina, Prof. Ana Breda and Prof. Tatiana Cordeiro from the University of Aveiro, and Prasoon Kumar Vinodkumar, a doctoral student from Tartu Ülikool. Daniela Dias, a student from the University of Aveiro, showcased her master's thesis on gamification in mathematics education, demonstrating the potential for making math lessons more engaging and interactive.

Teachers' perspective

VR in education

How do academic teachers evaluate the use of Virtual Reality in mathematics and other subjects?

"VR exercises in teaching science subjects include mathematics, physics, and other engineering issues. The ability to use elements of spatial mathematics and understand the surrounding world is a necessary condition for building competence in the field of engineering. Developing the ability to see 3D complex issues for some pupils or students is very difficult or even impossible."

"The use of VR in education fosters experiential learning, helping students and educators alike to visualize and interact with abstract concepts. It also promotes a hands-on, immersive experience that can be applied effectively across various subjects. The using of VR in education supports a new dimension of education, helps students and teachers visualize abstract concepts and thus improves their imagination in space. VR can also be used in other subjects and thus increases the interest of students in education."

"The use of VR exercises in mathematics learning was highly effective, allowing participants to visualize complex geometric concepts in 3D. Additionally, tasks introducing basic Oculus VR operations helped participants develop essential digital competences, further enhancing their learning experience."

"The use of VR in education fosters experiential learning, helping students and educators alike to visualize and interact with abstract concepts. It also promotes a hands-on, immersive experience that can be applied effectively across various subjects."

How do academic staff from the consortium evaluate the European project(s) such as the Math3DgeoVR project?

"International cooperation in projects such as Math3DgeoVR is invaluable. It not only enriches academic exchange moments but also provides participants with opportunities to develop cultural and socioemotional skills. Working with peers and mentors from diverse backgrounds fostered an opportunity for teamwork, adaptability, and a deeper appreciation of different perspectives."

"The international cooperation within the Math3DgeoVR project was both inspiring and creative. It facilitated the exchange of experiences between countries such as Estonia, Portugal, Slovakia, and Poland, while enabling the collaborative development of educational materials for students. This synergy enriched the project's outcomes and fostered cross-cultural understanding."

"International cooperation in European projects is extremely important in the field of building worldviews and exchanging experiences in the field of education.

Communicating only in the local environment allows for expanding competencies in the field of tutoring, mentoring, and modern teaching techniques in a narrow scope. Thanks to participation in international projects, it is possible to look at the teaching process in a much broader way."

Why is it valuable for students to participate in such events as the summer school in Aveiro?

"Participating in such events is valuable for students as it fosters the development of soft skills, such as presenting and collaborating in international settings. It enhances English language proficiency, facilitates the exchange of experiences across countries, and provides opportunities to co-create educational materials and apply them in practice. These experiences enrich students both academically and personally, preparing them for global challenges."

"For students, participation in summer school is extremely important due to the opportunity to get to know the culture of other countries, build new friendships, respect other people with different views and other values that sometimes differ significantly. Additionally, during summer schools, it is possible to combine rest with learning."

"For students, events such as summer schools go beyond academic learning. They enable participants to cultivate cultural awareness, build lasting connections, and develop essential soft skills, such as communication and collaboration, which are critical for personal and professional growth."

Why is it valuable for academic teachers to participate in such events as the summer school in Aveiro?

"For teachers, the summer school creates a unique space for professional development, offering exposure to innovative teaching tools and methodologies while also encouraging the exchange of ideas in a culturally diverse and collaborative environment. Participation in events such as the Aveiro Summer School allows teachers to exchange teaching methods in mathematics and gives a new perspective on their work. It also offers practical experience in solving technical challenges in unfamiliar technological environments, supports professional growth and strengthens international cooperation."

"It is an opportunity to meet students from other countries, have joint discussions with other teachers, exchange experiences in the field of teaching methods and also expand the network of people with whom it is possible to cooperate in the future"

"A unique space for professional development, offering exposure to innovative teaching tools and methodologies while also encouraging the exchange of ideas in a culturally diverse and collaborative environment."

"For teachers, it is highly valuable as it enables the exchange of teaching methods in spatial geometry and provides insights into working with diverse soft skills frameworks. It also offers practical experience in solving technical challenges within unfamiliar

technological environments, fostering professional growth and enhancing international collaboration."

Testing with students

A pilot testing session of innovative VR educational exercises was conducted with students from consortium universities. The session aimed to evaluate the effectiveness and user experience of the VR learning modules. **SUS (System Usability Scale)** was employed to measure user satisfaction and identify areas for improvement.

- **Purpose:** Evaluate VR educational exercises
- **Participants:** Students from consortium universities
- **Methodology:** Pilot testing, user experience assessment
- **Evaluation method:** SUS (System Usability Scale)

Valuable feedback from both students and teachers was collected, leading to significant improvements in the design and functionality of the VR exercises

Multiplier Events

The project's multiplier events targeted both students and educators, providing hands-on exposure to VR technology in teaching and learning. Events included presentations, workshops, and knowledge transfer sessions held across partner institutions. For instance, the Slovak-Czech Conference on Geometry and Graphics in September 2022 served as a platform for presenting project achievements and sharing best practices. During these events, six key VR modules were demonstrated, drawing attention to their potential for revolutionizing mathematical education.

VR Modules in Focus

The Math3DGeoVR project developed and tested a variety of VR modules to address different aspects of geometry and mathematics. These modules not only enriched the learning experience but also revealed the unique advantages and limitations of VR in education.

Key Highlights

Visualization and Engagement: Modules such as “Angles in Prisms” and “Systems of Linear Equations” were praised for their clarity and ability to transform abstract concepts into tangible, 3D visualizations. Participants found the immersive experience particularly motivating and engaging.

Challenges: The “Non-Euclidean Geometry” module, though groundbreaking, posed challenges such as motion sickness and difficulties in intuitive navigation. Participants suggested improvements in camera movement and interface design to enhance usability.

Suggestions for Improvement

Based on feedback:

- Tutorials should be more interactive, with clearer instructions for tasks like drawing angles or manipulating objects in 3D.
- Modules should include additional labels and annotations to aid understanding.
- Improvements in video quality and smoother transitions between tasks were recommended.
- Expanding the module library with new topics like derivatives, cross-sections of solids, and integrals was highly encouraged.

Dissemination and Outreach

The Math3DGeoVR project was widely promoted through conferences, social media, and institutional events. Notable dissemination efforts included:

- A presentation during the Open Day at the University of Žilina, attracting over 100 participants.
- Social media campaigns and website updates that extended the project’s reach to diverse audiences.
- Contributions to international conferences, showcasing the project’s impact on geometry education.

Conclusion

The Math3DGeoVR project exemplifies the transformative power of VR in education. By providing an immersive and interactive platform, it has enabled learners to grasp complex mathematical concepts with ease. While challenges remain in optimizing technology and accessibility, the project’s success lays a strong foundation for future innovations in VR-based education. With continued funding and collaboration, Math3DGeoVR is poised to redefine how mathematics is taught and learned across diverse educational contexts.